

TaMaDi
Table Maker's Dilemma
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<http://perso.ens-lyon.fr/jean-michel.muller/TaMaDi.html>



Mercredi 27 octobre

- ▶ 10h30 : brève présentation du projet (J.M. Muller)
- ▶ 11h00 : présentation ANR (N. Nadah) ;
- ▶ 11h30 : repas ;
- ▶ 13h30 : SLZ for dummies (G. Hanrot) ;
- ▶ 14h30 : Solutions de calcul (P. Fortin, S. Collange, S. Torres) ;
- ▶ 16h00 : La preuve formelle pour les nuls (L. Rideau et/ou L. Théry) ;
- ▶ 17h00 : Une expérience autour du Lemme de Hensel (E. Martin Dorel).

20h00 : repas au restaurant Kiozen, 14 rue d'Ambroise, 69002 Lyon (métro Bellecour).

- ▶ 9h15 : Les "Hardest to round cases" obtenus jusqu'ici (V. Lefèvre);
- ▶ 9h45 : LLL for dummies (G. Hanrot);
- ▶ 10h45 : Calcul de fonctions avec arrondi correct : traitement d'un exemple issu de CRLibm (C. Lauter);
- ▶ 11h45 : repas
- ▶ 14h : autour de la norme infinie (M. Joldes);
- ▶ 15h : Some possible improvements to SLZ (A. Novocin);
- ▶ 15h30 : Discussion générale – synthèse – programme de travail pour les mois à venir.

Presentation of the TaMaDi project

System	$\sin(10^{22})$
exact result	$-0.8522008497671888017727\dots$
HP 48 GX	-0.852200849762
HP 700	0.0
HP 375, 425t (4.3 BSD)	$-0.65365288\dots$
matlab V.4.2 c.1 for Macintosh	0.8740
matlab V.4.2 c.1 for SPARC	-0.8522
SPARC	-0.85220084976718879
IBM RS/6000 AIX 3005	$-0.852200849\dots$
DECstation 3100	NaN
Casio fx-8100, fx180p, fx 6910 G	Error
TI 89	Trig. arg. too large

Avoiding this : correct rounding.

Correct rounding

In the IEEE 754 standard, the user defines an *active rounding mode* (or *rounding direction attribute*) among :

- ▶ **round to the nearest** (default) in case of a tie, value whose integral significand is even ;
- ▶ **round towards $+\infty$** .
- ▶ **round towards $-\infty$** .
- ▶ **round towards zero**.

A correctly-rounded operation whose entries are FP numbers must return what we would get by infinitely precise operation followed by rounding.

Correct rounding

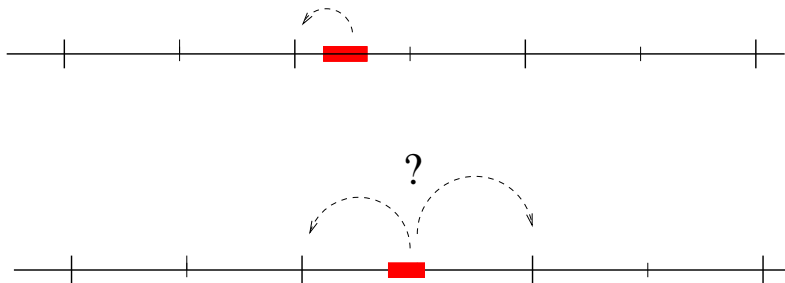
IEEE-754 (1985) : **Correct rounding** for $+$, $-$, \times , \div , $\sqrt{\quad}$ and some conversions. Advantages :

- ▶ if the result of an operation is exactly representable, we get it ;
- ▶ if we just use the 4 arith. operations and $\sqrt{\quad}$, deterministic arithmetic : one can elaborate **algorithms** and **proofs** that use the specifications ;
- ▶ accuracy and portability are improved ;
- ▶ playing with rounding towards $+\infty$ and $-\infty \rightarrow$ **certain** lower and upper bounds.

FP arithmetic becomes a **structure in itself**, that can be studied.

IEEE-754 (2008) : suggests correct rounding for some elementary functions.

The Table Maker's Dilemma



By the end of the project, we aim at producing...

- ▶ binary64/double precision (and possibly, double-extended) and decimal64 : **HR cases** for all the functions of a standard C math library (will x^y and some trigs of large arguments be reachable?);
- ▶ the **certificates** proving the validity of these cases will be freely available too;
- ▶ larger formats : for the most common functions, a value ϵ s.t. if we approximate the function with relative accuracy ϵ , rounding approximation \Leftrightarrow rounding exact value + certificates;
- ▶ **open-source software**, as portable as possible, for computing HR cases or accuracy bounds for other formats and functions and generating the certificates;
- ▶ **publication** in journals and/or proceedings of international conferences of algorithms, properties, etc.