Vers une formalisation du L-algorithme en Coq

Motivations

Towards solving the Table Maker's Dilemma on GPU

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***Abstract*—Since 1985, the IEEE 754 standard defines formats, rounding modes and basic operations for floating-point arithmetic. In 2008 the standard has been extended, and recommendations have been added about the rounding of some elementary functions such as trigonometric functions (cosine, sine, tangent and their inverses), exponentials, and logarithms.**

This isolation is efficiently performed using local affine approximations of the targeted function. Stehlé, Lefèvre and Zimmermann extended this method in 2003 [2], [3] (SLZ algorithm) for higher degree approximations, using the Coppersmith method for finding small roots of univariate modular

Plan

Plan

Ce qui a été fait

Plan

Ce qui a été fait

Ce qui reste à faire

Théorèmes des 3 distances

THE THREE GAP THEOREM
(STEINHAUS CONJECTURE)

TONY VAN RAVENSTEIN

(Received 18 November 1986; revised 6 August 1987)

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Abstract

This paper is concerned with the distribution of N points placed consecutively around the circle by an angle of α. We offer a new proof of the Steinhaus Conjecture which states that, for

Théorèmes des 3 distances

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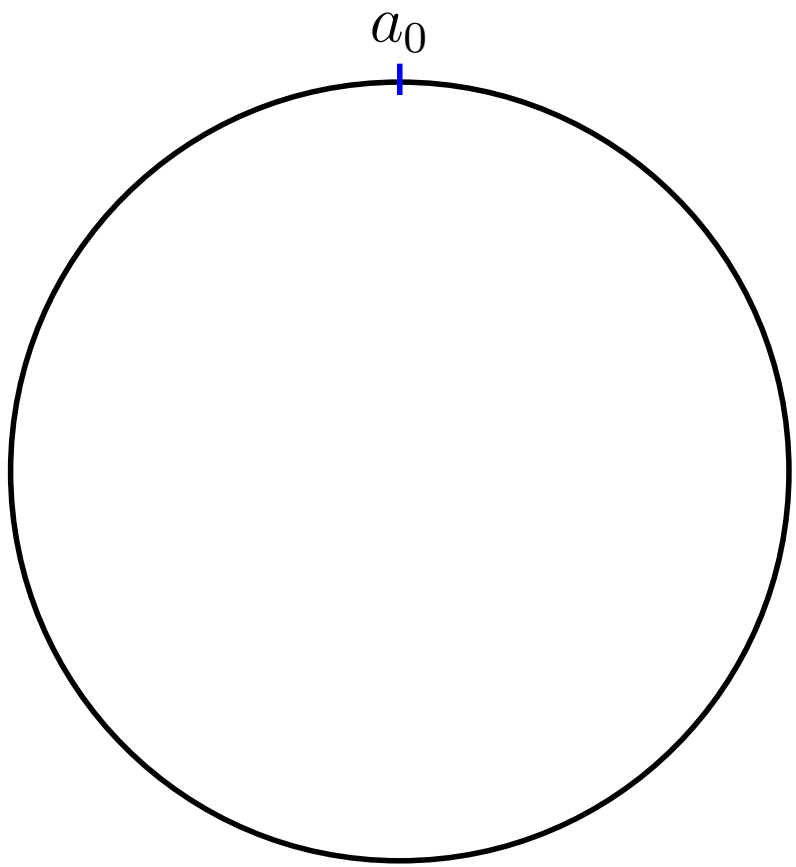
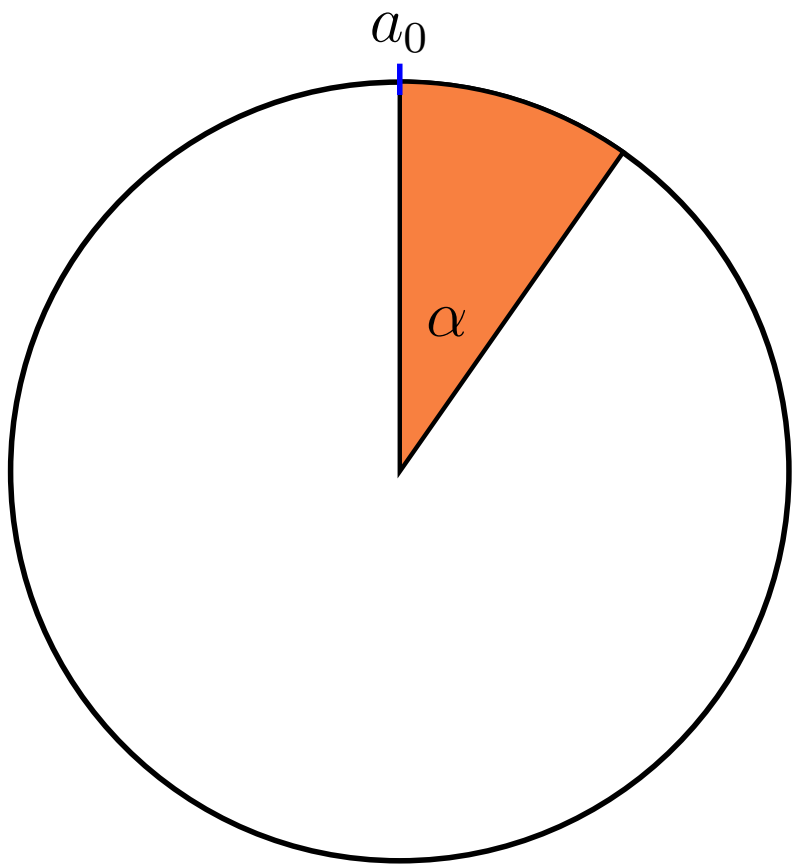


Théorèmes des 3 distances



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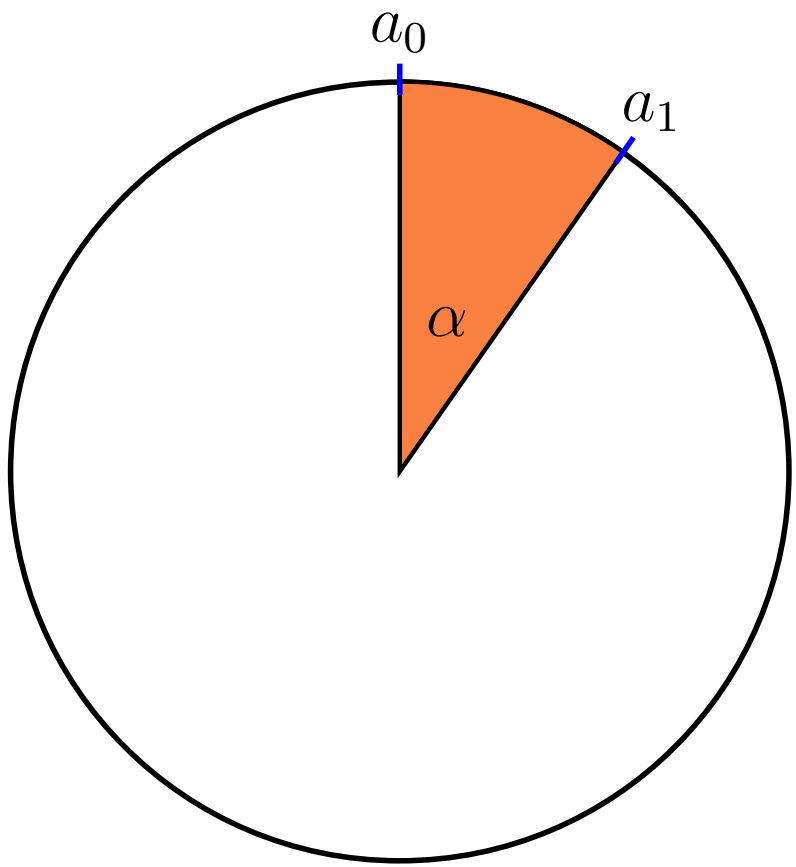

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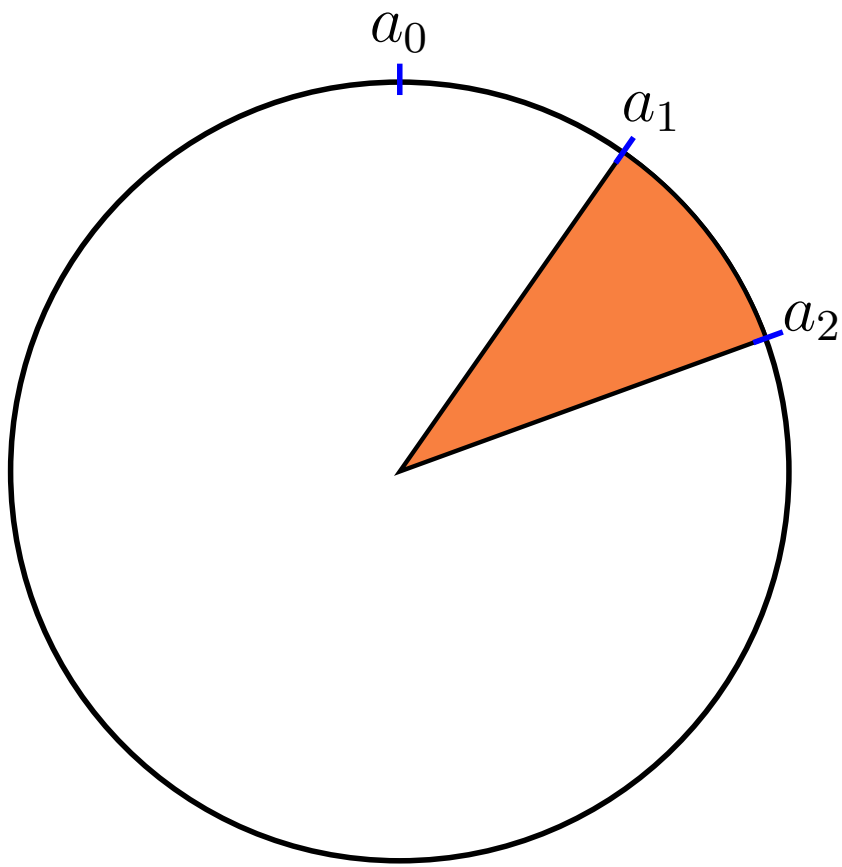

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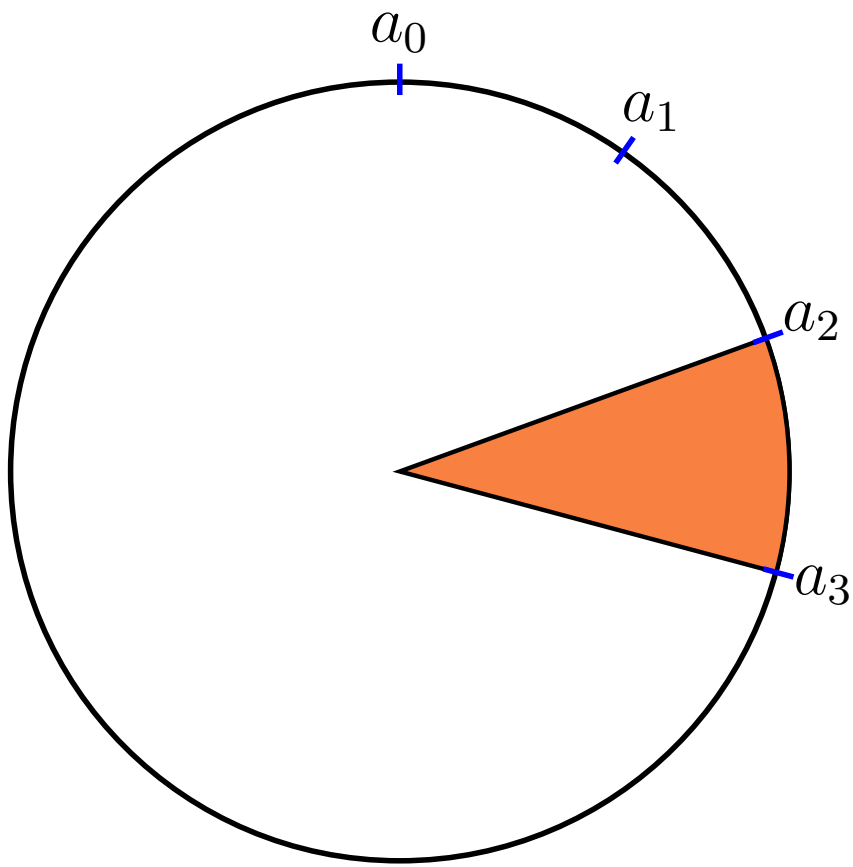

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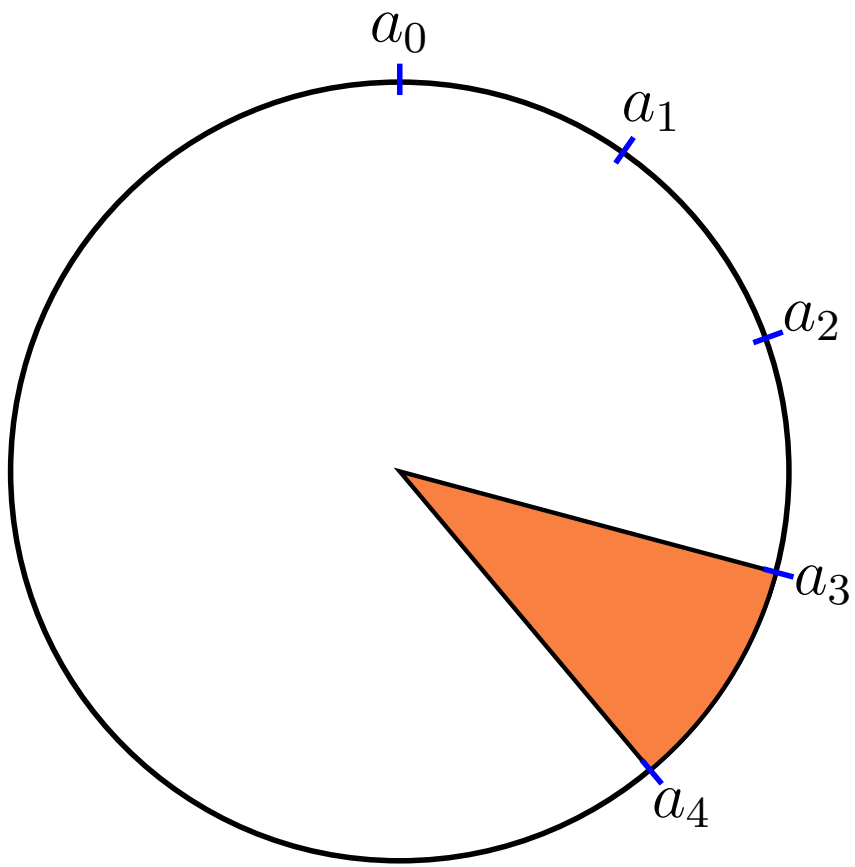

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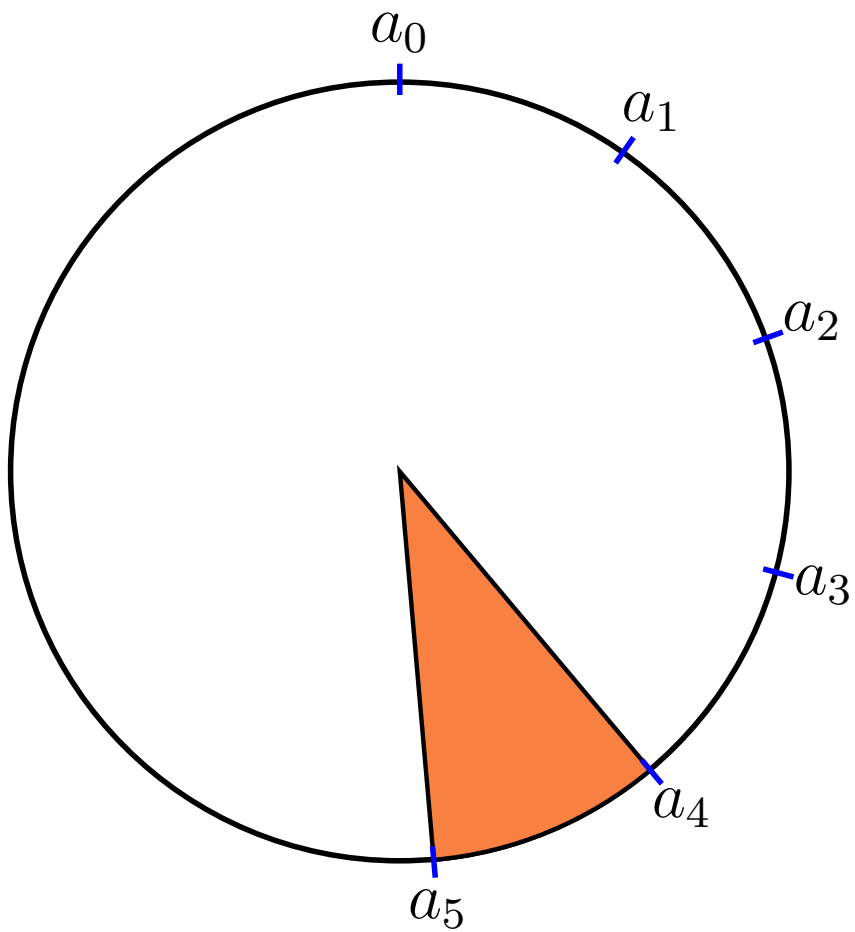

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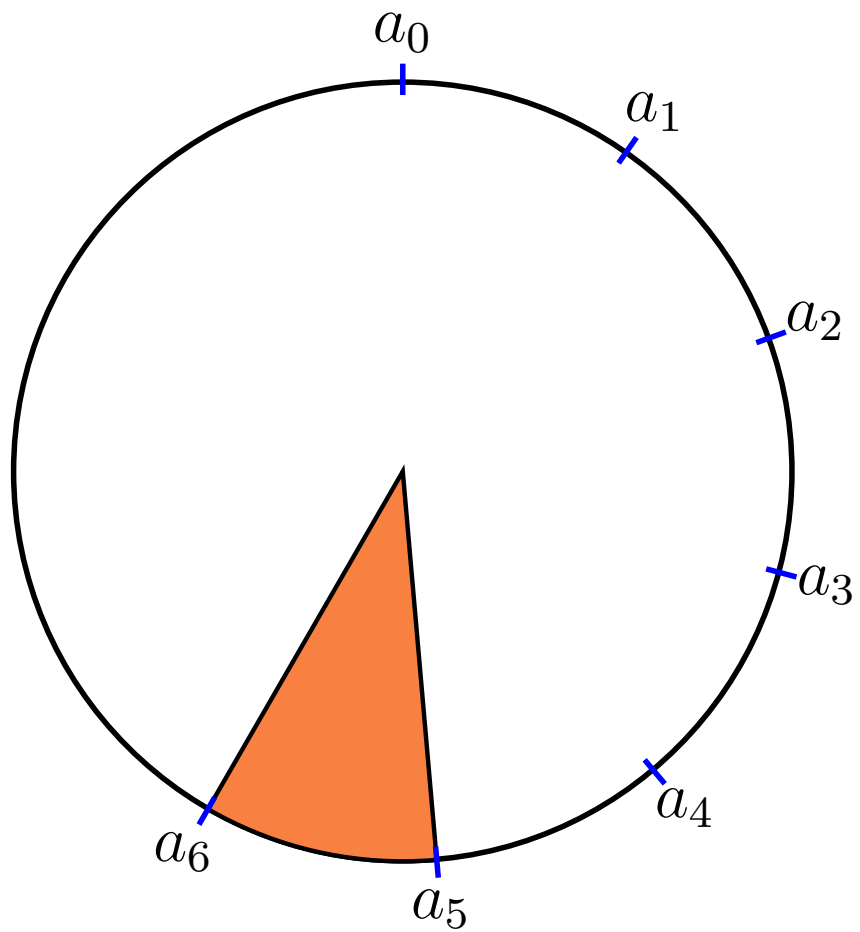

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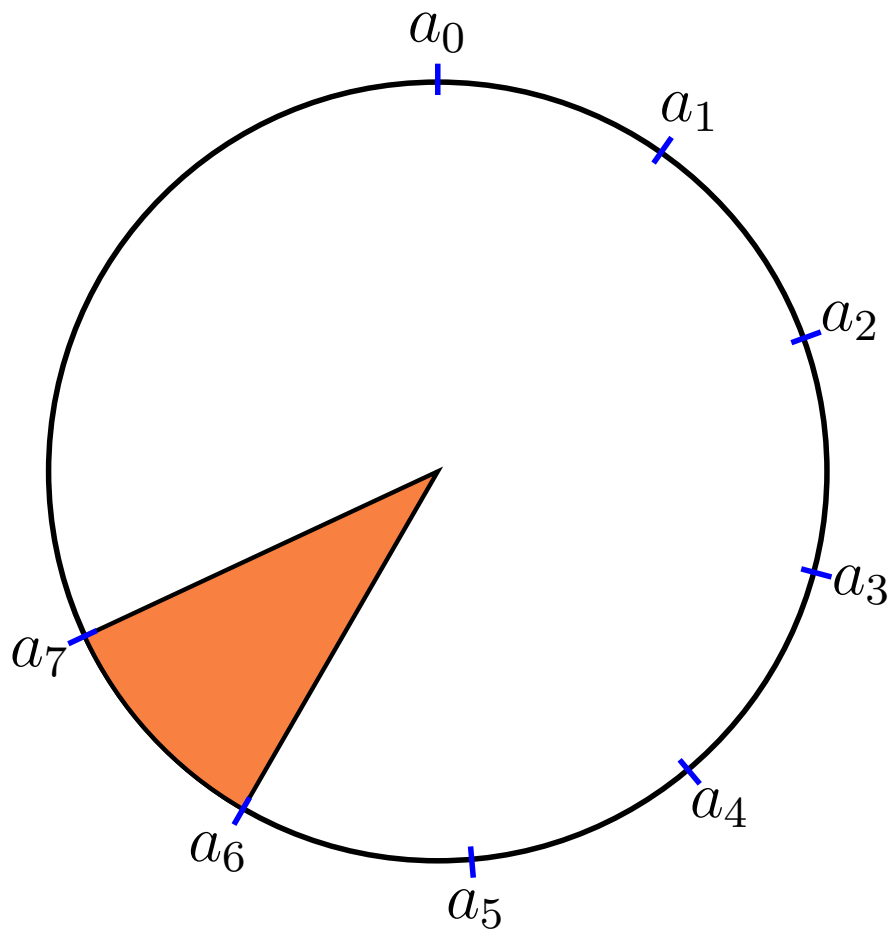

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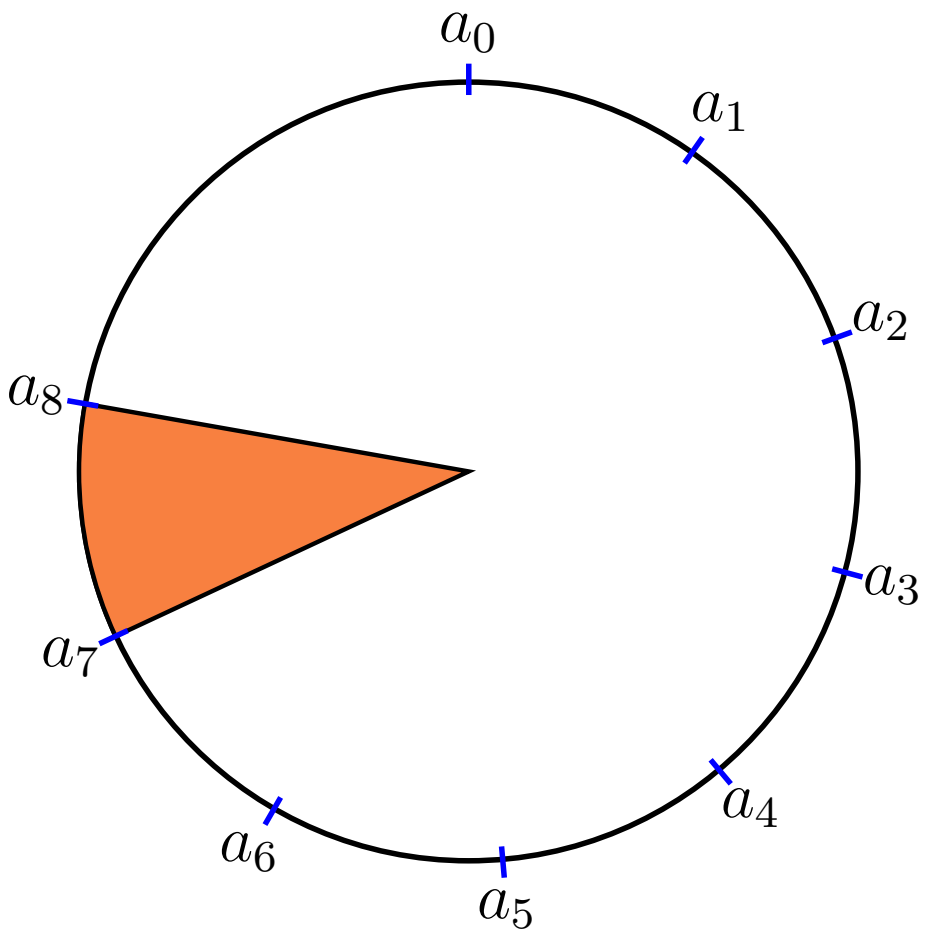

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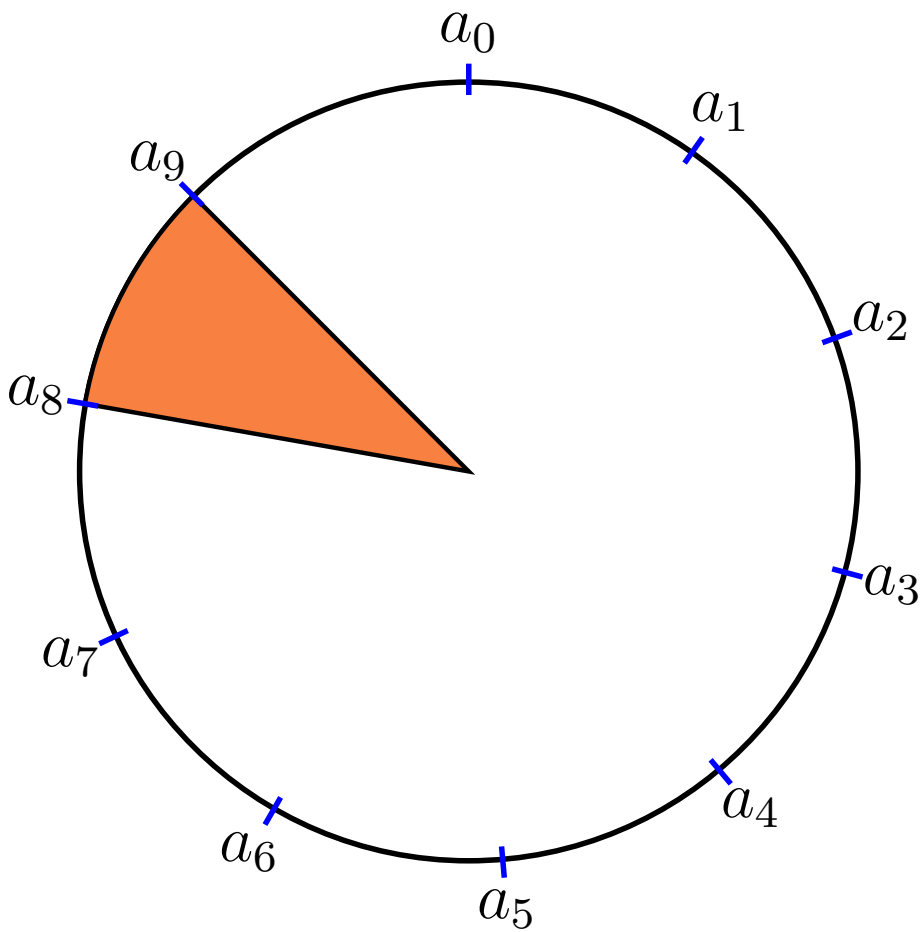

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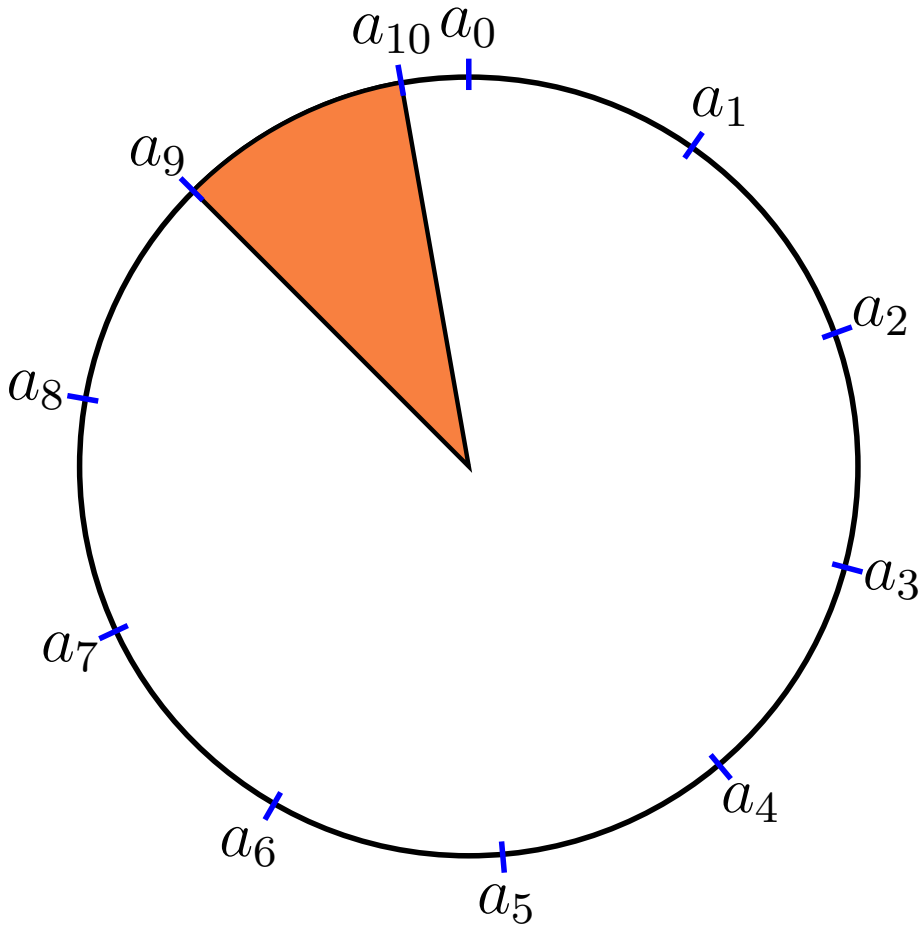

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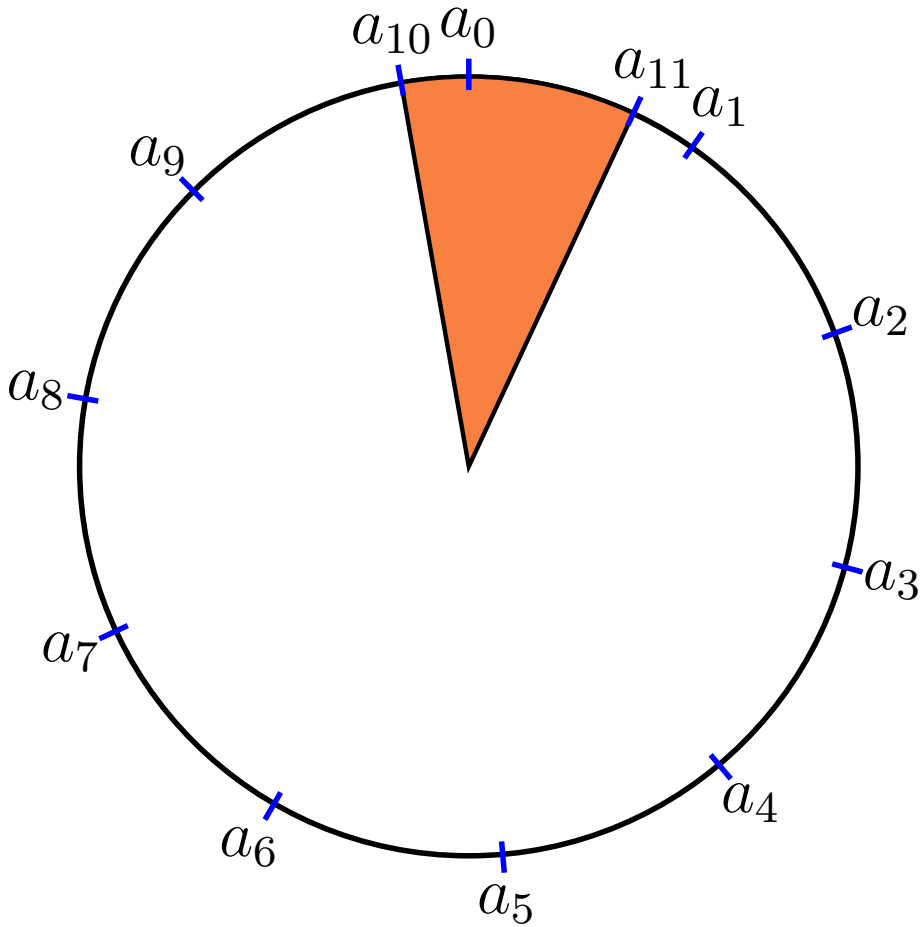

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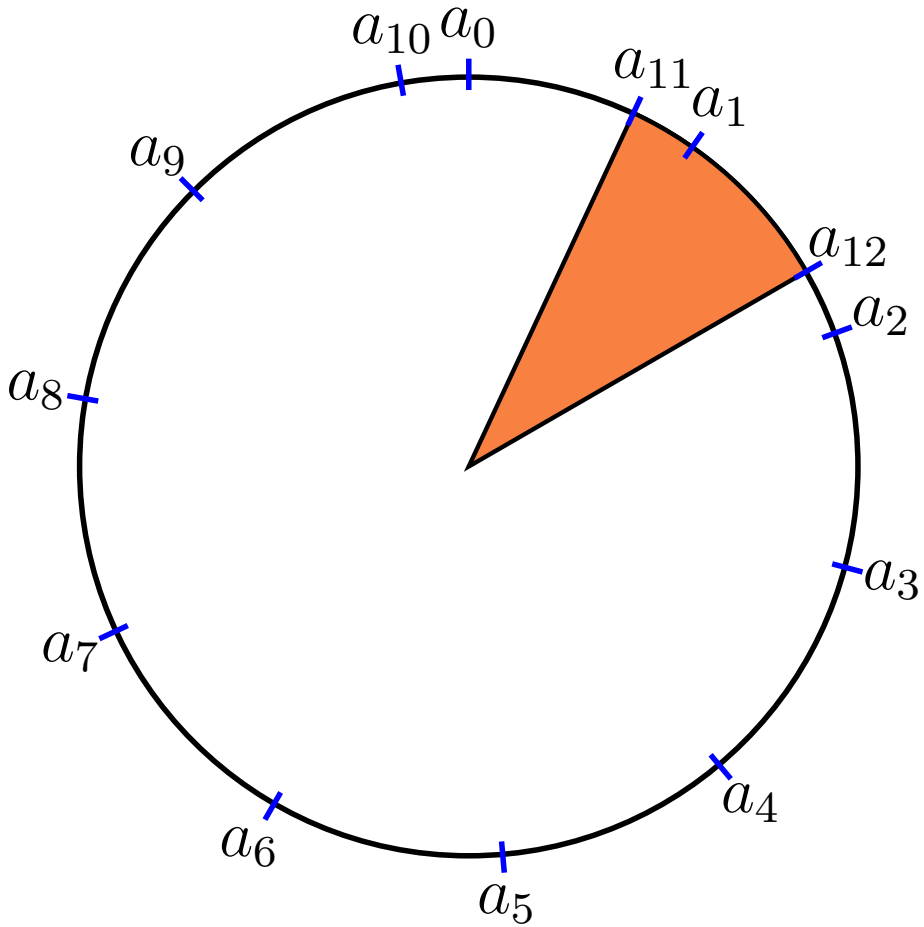

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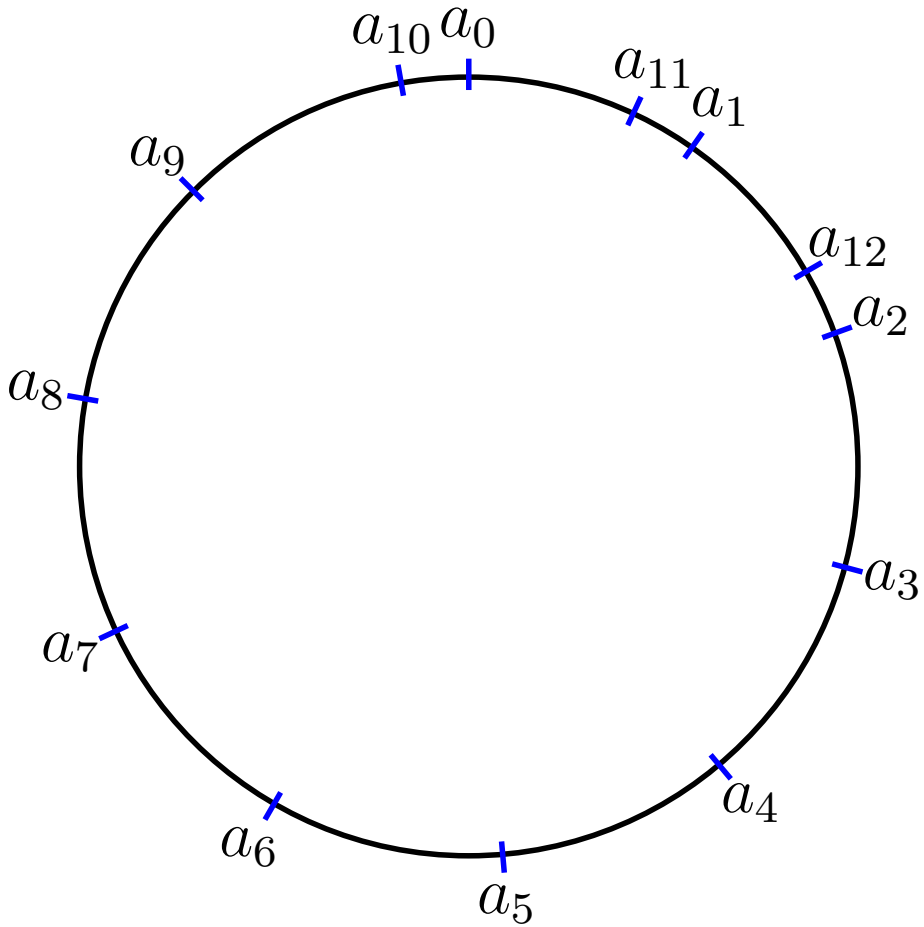

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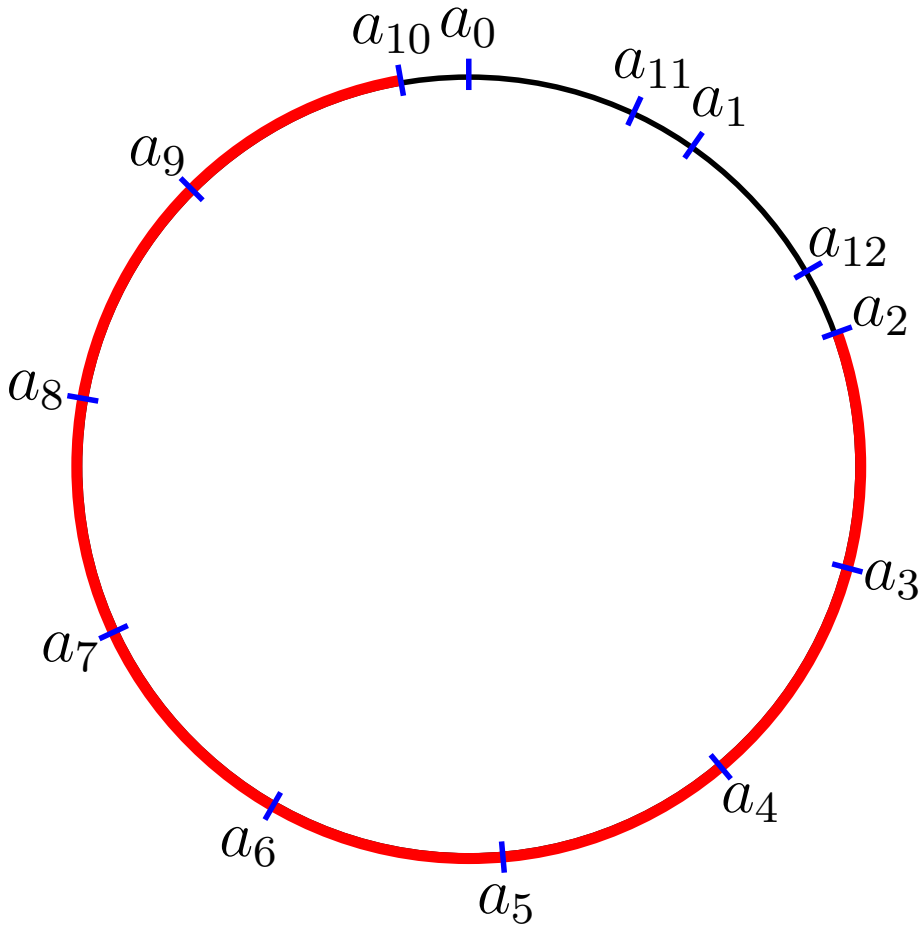

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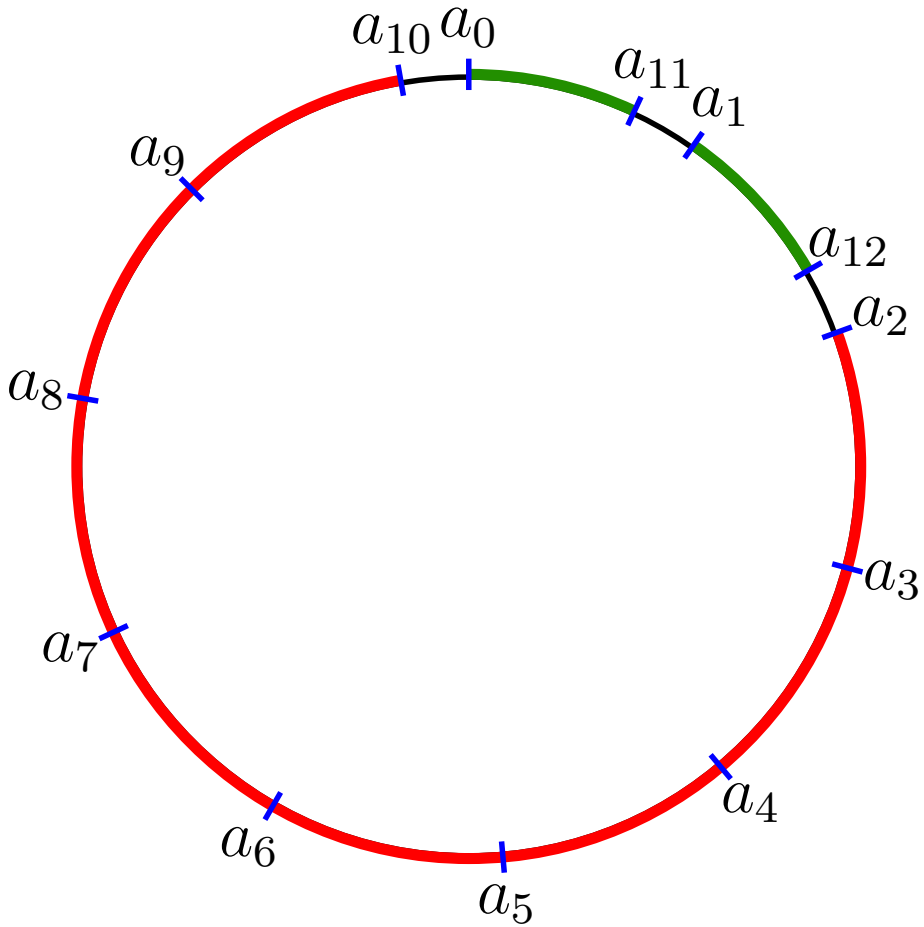

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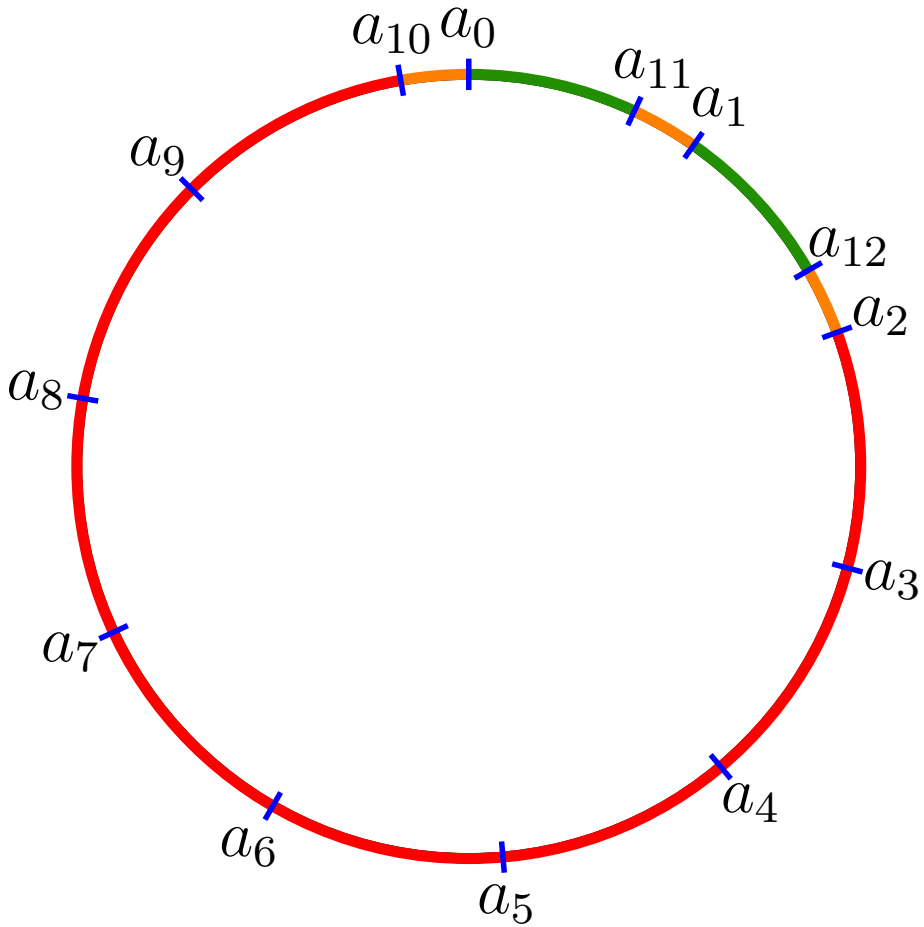

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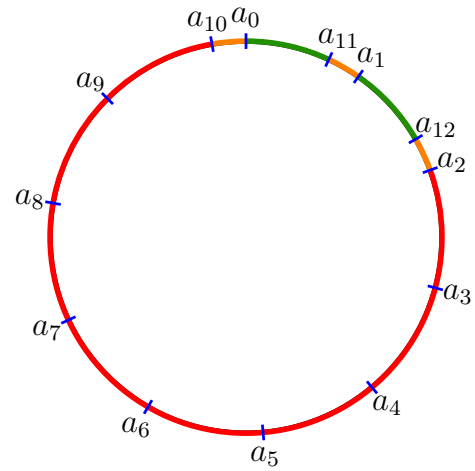
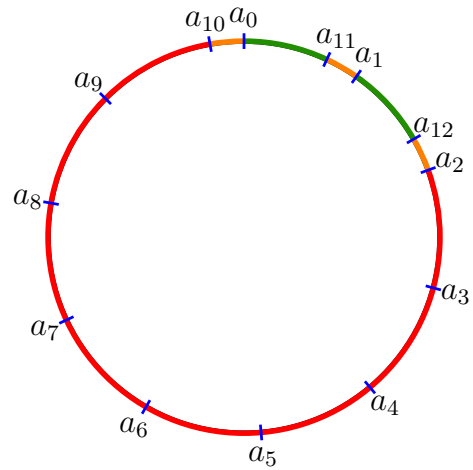


Théorèmes des 3 distances

$\alpha \in \mathbb{R}$, $N \in \mathbb{N}, a_n = \{\alpha n\}$, $a_n \neq 0$ **pour** $1 \leq n \leq N$



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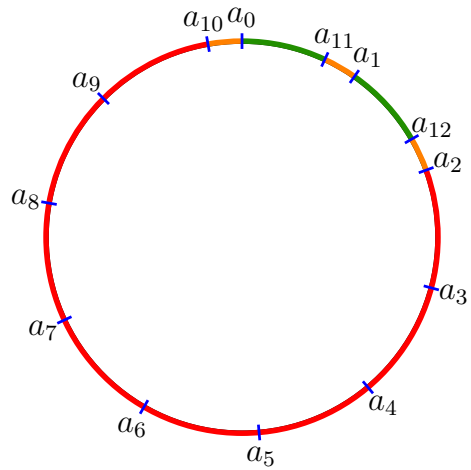

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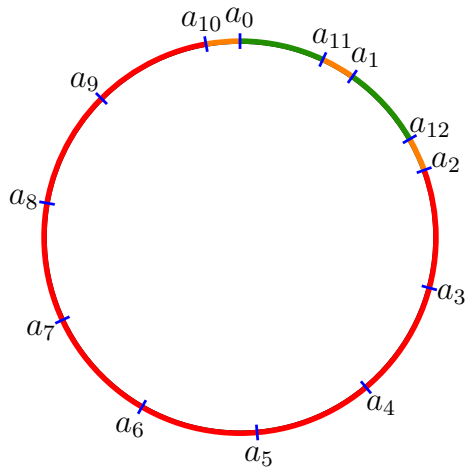
$\overline{12} = 2, \underline{12} = 1, m = 11, M = 10$



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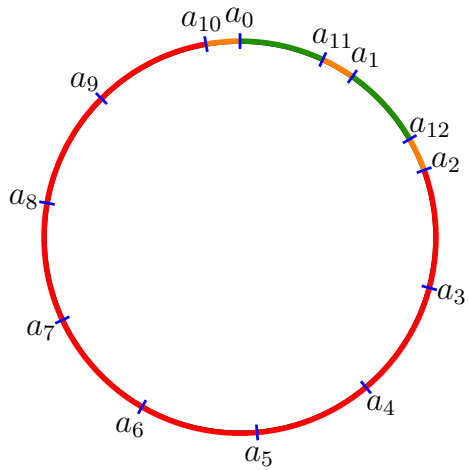
Si $n \leq N - m, \overline{n} = n + m,$ $\quad d = a_m$



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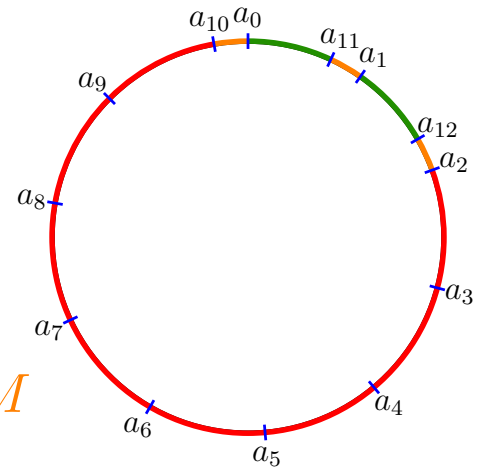
Si $M < n \leq N, \overline{n} = n - M \quad d = 1 - a_M$



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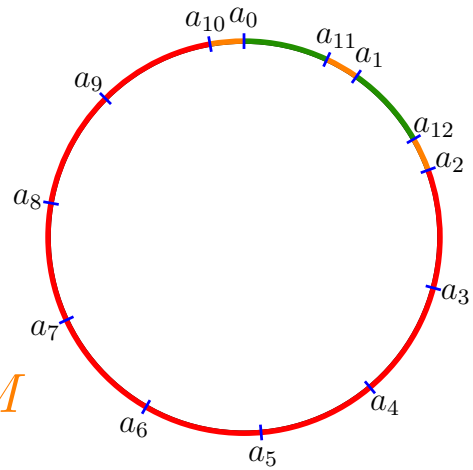
Si $N - a < n \leq M, \overline{n} = n + m - M$ $\quad d = 1 + a_m - a_M$



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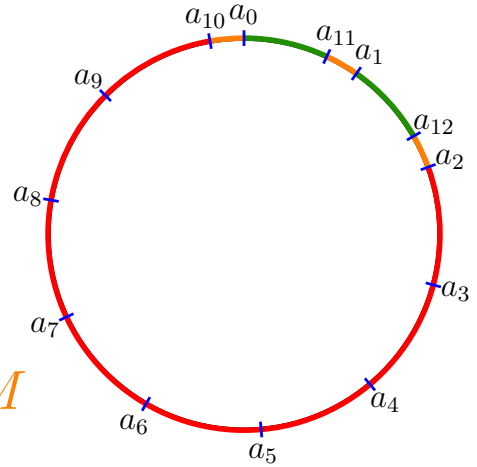

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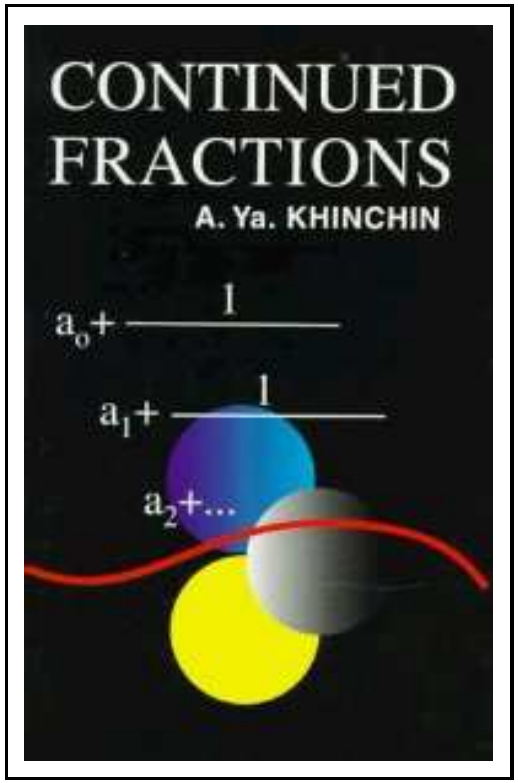
Si $N - a < n \leq M, \overline{n} = n + m - M \quad d = 1 + a_m - a_M$

$[m\alpha]/m < \alpha < (1 + [M\alpha])/M$, $m(1 + [M\alpha]) - [m\alpha]M = 1$

Fraction continue



Fraction continue

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Médian

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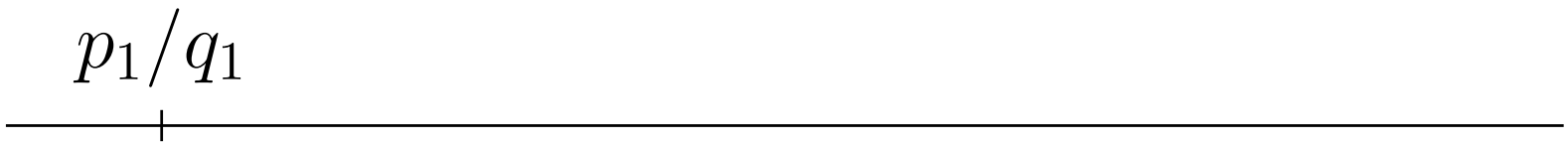
Médian



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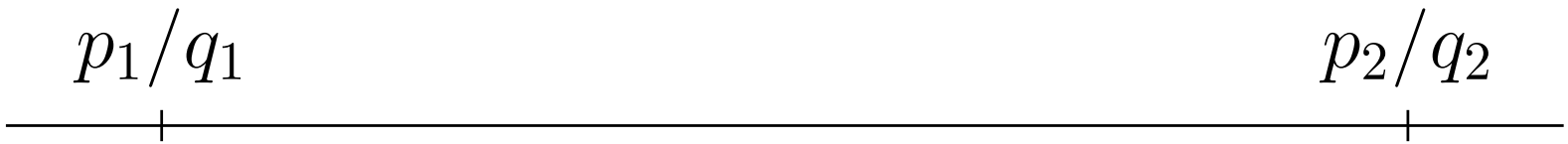

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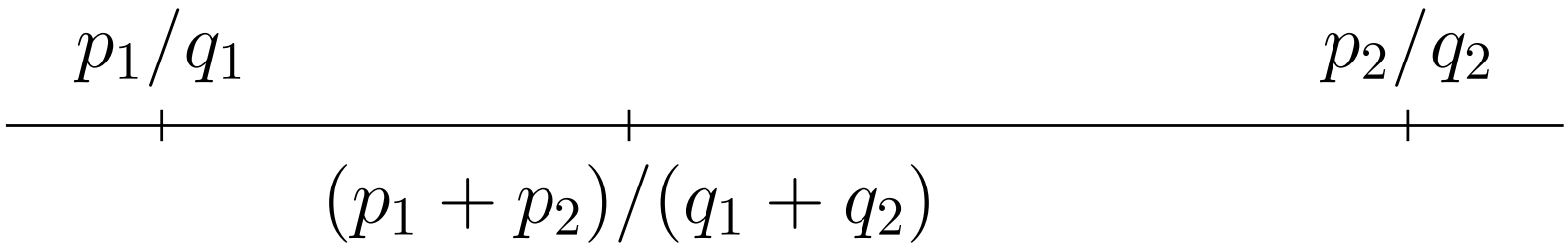

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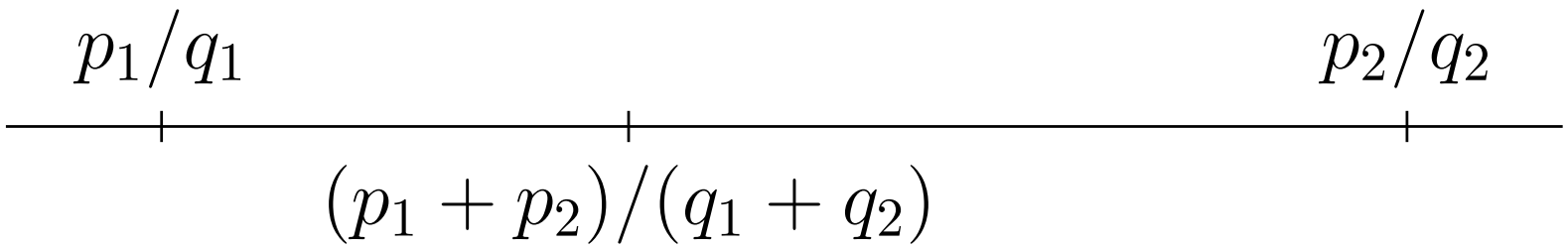
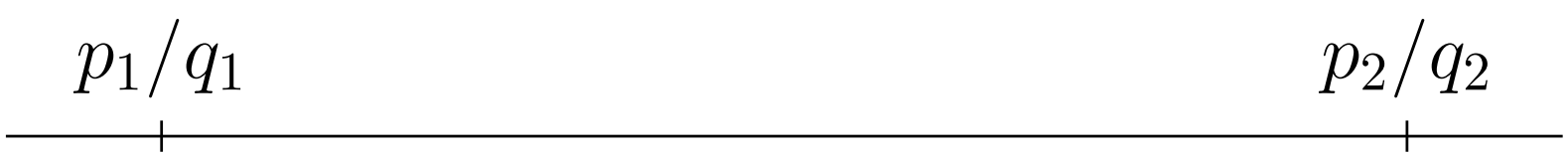


Approximation



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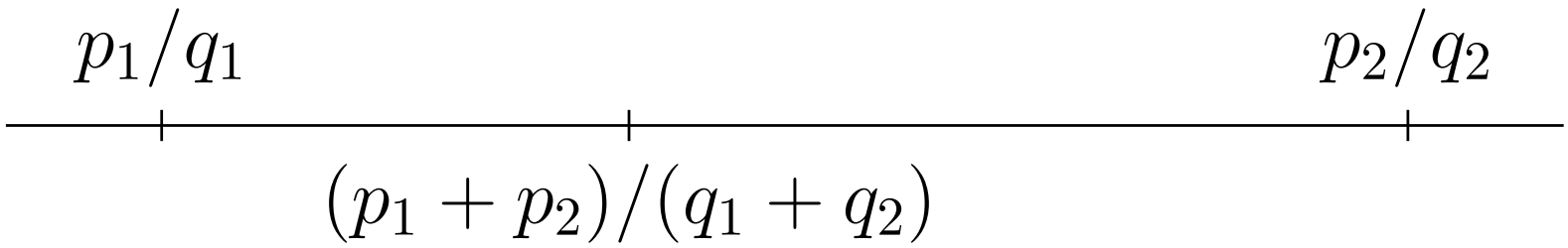

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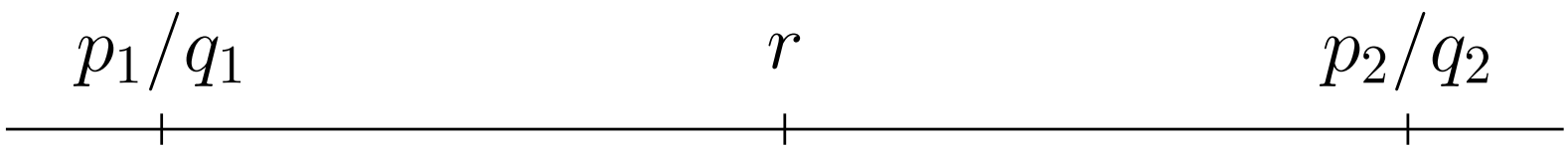

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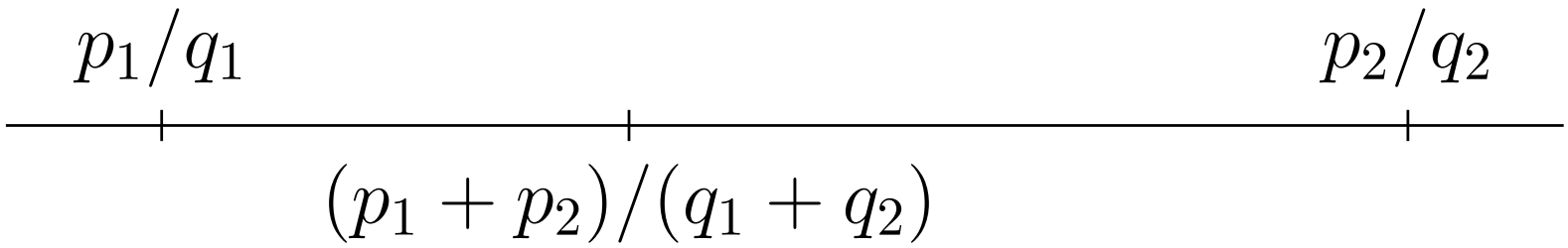

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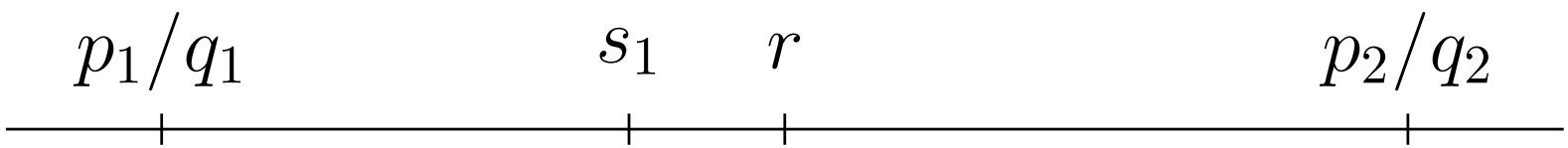

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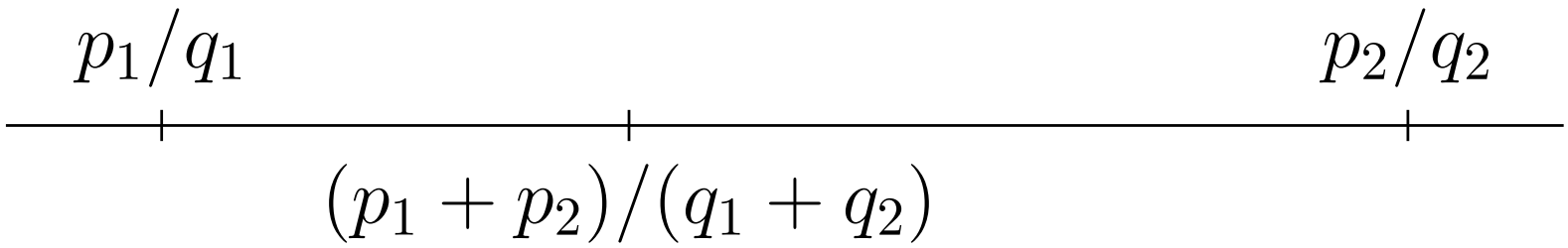

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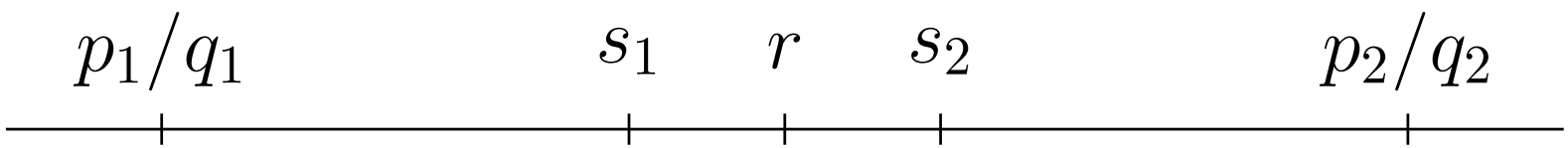

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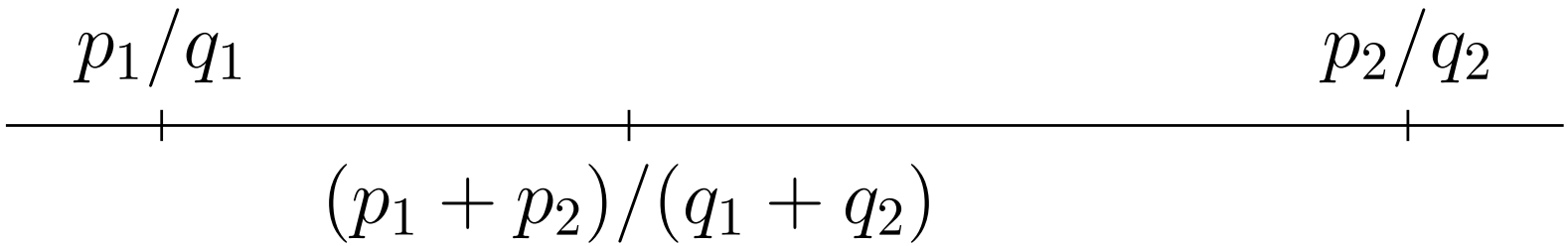

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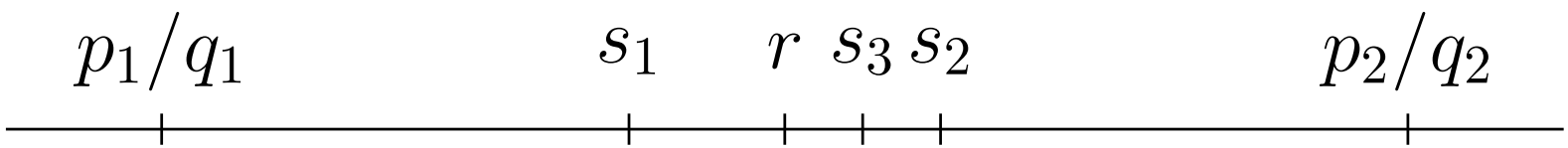

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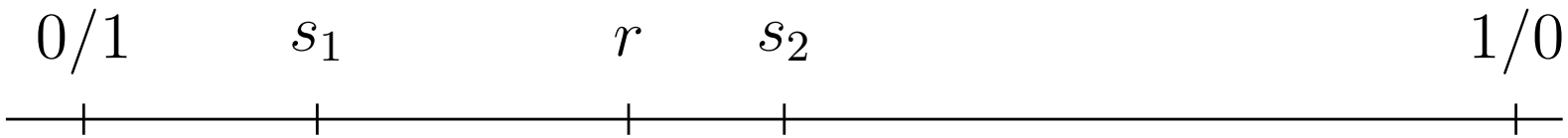
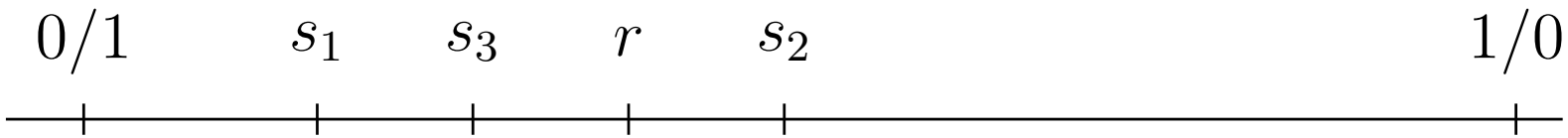
$s_1 = a_0$

Fraction continue



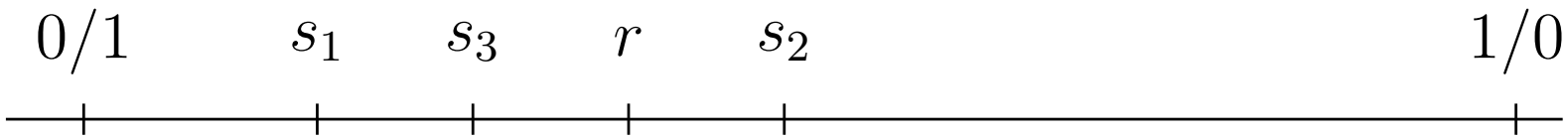
$s_1 = a_0$, $s_2 = a_0 + 1/a_1$

Fraction continue



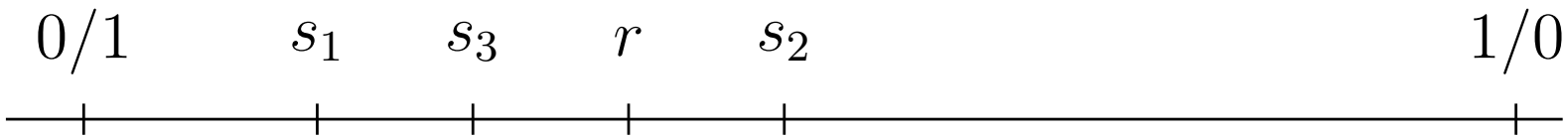
$s_1 = a_0$, $s_2 = a_0 + 1/a_1$, $s_3 = a_0 + 1/(a_1 + 1/a_2)$

Fraction continue



$s_1 = a_0$, $s_2 = a_0 + 1/a_1$, $s_3 = a_0 + 1/(a_1 + 1/a_2)$, ...

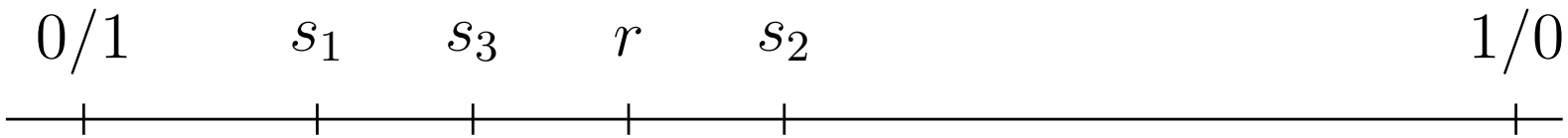
Fraction continue



$s_1 = a_0$, $s_2 = a_0 + 1/a_1$, $s_3 = a_0 + 1/(a_1 + 1/a_2)$, ...

$r \rightsquigarrow [a_0; a_1; a_2; \ldots]$

Fraction continue

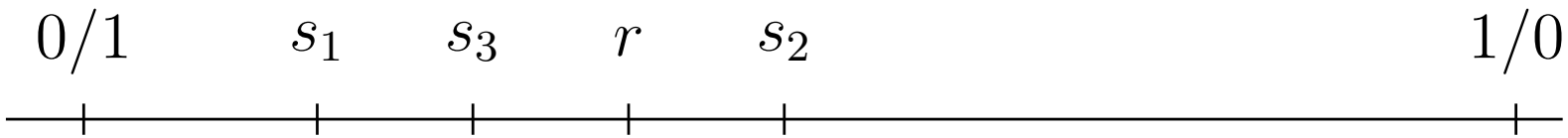


$s_1 = a_0$, $s_2 = a_0 + 1/a_1$, $s_3 = a_0 + 1/(a_1 + 1/a_2)$, ...

$r \rightsquigarrow [a_0; a_1; a_2; \ldots]$

convergent $s_n = p_n/q_n$

Fraction continue



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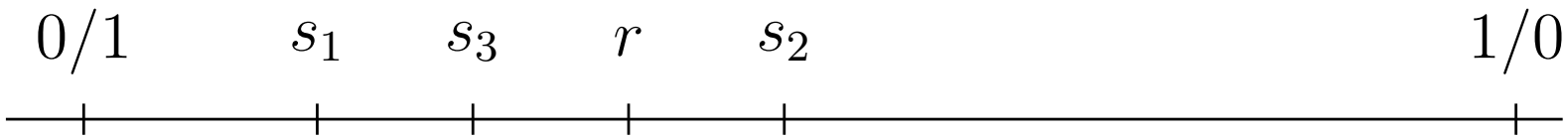

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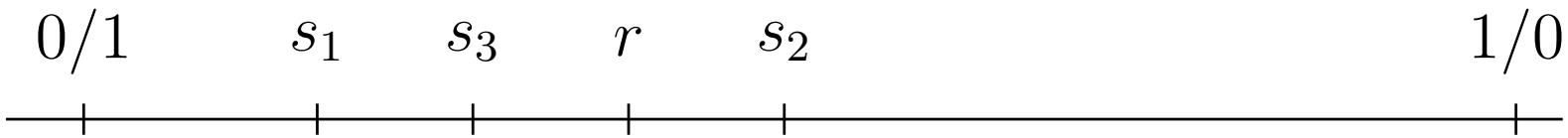

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convergent interm. $s_{n,i} = (ip_{n+1} + p_n)/(iq_{n+1} + q_n)$, $0 \leq i \leq a_{n+1}$

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Ce qui reste à faire

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$\texttt{inf}\{b - ax \mod 1| \leq N\} < \epsilon$?

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